A Pascal Graph Property and Computer Network Topology

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Abstracts:

It has always been tried to represent the different computer network topologies using appropriate graph models. Graphs have huge contributions towards the performance improvement factor of a network. Some major contributors are de-Bruijn, Hypercube, Mesh and Pascal. They had been studied a lot and different new features were always a part of research outcome. As per the definition of interconnection network it is equivalent that a suitable graph can represent the physical and logical layout very efficiently. In this present study Pascal graph is researched again and a new characteristics has been discovered. From the perspective of network topologies Pascal graph and its properties were first studied more than two decades back. Since then, a numerous graph models have emerged with potentials to be used as network topologies. This new property is guaranteed to make an everlasting mark towards the reliability of this graph to be used as a substantial contributor as a computer network topology. This shows its credentials over so many other topologies. This study reviews the characteristics of the Pascal graph and the new property is established using appropriate algorithm and the results.

Keywords: Pascal Graph; Pascal Matrix; Dependable node of Pascal Graph; Computer Network Topology.

1. Introduction

The ontogeny of Pascal Graph (PG) was Pascal Matrix (PM) that in turn was generated meticulously from Pascal’s triangle [1]. Scientist have been putting a lot of efforts in ameliorating computer network properties [4].

Wide varieties of graph model worked as resource to their brainstorming contribution in this field [3]. Pascal graph is one of those resources and played a significant role as soon as it exploration was initiated almost two decades back. As a consequence, several similar graph models emerged in this arena with laudable potentials to be used as computer network topologies [2]. The inspiration of this study stemmed from consistent urge to contribute substantially in this crescendo of research from the perspective of computer network topology. Here we have reincarnated Pascal Graph with another feather added to its credentials. We have reviewed to its characteristics and established a significant property with adequate theoretical practical support.

2. Genesis of Pascal Graph:

We introduced Pascal Graph from historical modern perspective. Blaise Pascal (1623 - 1662) first conceptualized Pascal’s Triangle around the middle of seventeenth century [7]. This Pascal triangle played the most important role while generating Pascal Matrix [1].

2.1 Pascal Matrix Definition:

An (n x n) symmetric binary matrix is called the Pascal Matrix PM(n) of order n if its main diagonal entries are all 0’s and its lower (and therefore the upper also) consists of the first (n-1) rows of Pascal Triangle modulo 2. Where pm_{i,j} denotes the element of i^{th} row and j^{th} column of the Pascal Matrix [1].

2.2 Pascal Graph Definition:

An undirected graph of n vertices corresponding to PM(n) as an adjacency matrix is called Pascal Graph (n),
where \( n \) is the order of the Pascal graph \([1]\). An example of Pascal graph having 4 nodes is shown hereunder, for better comprehension of the study.

![Pascal Graph](image)

**2.3 Some Properties of Pascal Graph:**

There are certain pragmatic properties that make Pascal graph a better choice for a computer network topology over many others. Some of those properties are given below:

- \( PG(n) \) is a subgraph of \( PG(n+1) \) \( \forall n \geq 1 \).
- All Pascal Graph \( PG(i) \) \( i \leq 1 \leq 7 \) are planner; all Pascal Graph of higher order are non-planner.
- Vertex \( V_i \) is adjacent to all other vertices in the Pascal Graph. Vertex \( V_i \) is adjacent to \( V_{i+1} \) in the Pascal graph for \( i \geq 1 \).
- All Pascal Graph of order \( \geq 3 \) are 2-connected.
- No two even no of vertices of a Pascal Graph are adjacent.
- There are at least two edge disjoint path of length \( \leq 2 \) between any two distinct vertices in \( PG(n) \), \( 3 \leq n \).

**2.4 Algorithm of Pascal Graph:**

*Step 1:* Enter \( n \) number of vertices.

*Step 2:* Initialize \( LT[n,0] = 1 \);

*Step 3:* From the lower left triangle \([LT(n,n)]\) by adding the number directly above and to the left with the number directly above and to th right to find the new value. If either the number to the right or left is not present, substitute a zero to its place.

*Step 4:* From the upper right triangle \([UT(n,n)]\) using the same manner.

*Step 5:* Convert the lower left triangle into binary values.

*Step 6:* Convert the upper right triangle into binary values.

*Step 7:* From the final adjacency matrix \([PM(n,n)]\) of with \( LT(n,n) \) and \( UT(n,n) \).

*Step 8:* Stop.

**3. Significance of the Work:**

It is better not to predestine any claim before underpinned with adequate reasoning. To bolster this, the basic requirement must be articulated and hence substantiated. Studying different graph with respect to their properties has been a basic research interest for a long time \([6]\). It can be said without any ambiguity that there are certain criteria in any network topology that are given priorities to judge its credentials. Those criteria are, (i) degree of the network, (ii) diameter of the network, (iii) scalability of the network and (iv) reliability of the network \([2]\). There is a tradeoff between degree related to hardware cost and diameter related to transmission time of messages within any Interconnection Network \([2]\). Reliability is the quality to sustain faulty circumstances and scalability is the scope of up-gradation without much alteration. To prove its worth, Pascal; graph have been merged cogently with Hypercube graph to engender simple-(q,n)-graph \([2]\). The destiny of the research is signified keeping this philosophy in mind.

**4. An Intriguing Characteristic of Pascal Graph as DNP:**

In this work a new property of Pascal graph has been characterized that has increased the reliability of the graph without perturbing its degree, diameter as well as scalability as computer network topology. As we mentioned in property (iii) in properties of Pascal graph, vertex \( v_1 \) of any Pascal graph is adjacent to all other vertices in that graph. It has been shown that if the index start from 0 instead of 1 then it can be proved that vertex \( v_0 \) in any Pascal graph is adjacent to all other vertices in that graph. Vertex \( v_1 \) is adjacent to \( v_{i+1} \) in the Pascal graph if \( i \geq 0 \)[2]. Extending the property (iii) in the properties of Pascal graph, it can be very well shown that there exist other node \( v_i \) (\( i \geq 0 \)) having the same property like \( v_1 \). According to property (iv), this node will never have even number as its index.
4.1 Thought of Work:

If we consider a Pascal graph containing n nodes i.e. PG(n) then by default v₀ (i ≥ 1) is adjacent to all other nodes [1]. The exploration does not conclude with it. We have found and recommended other node similar to v₀ in the PG(n) and termed it as Dependable Node of Pascal Graph (DNP); the special node, other than node v₀, of Pascal graph PG(n) with same degree like v₀.

To satiate our claim and to rationalize the whole thing we have used very simplistic approach. The explanation with some suitable example is given below.

Case 1: \( n = 2^{\log n} \)

Case 2: \( n \ll 2^{\log n} \)

Case S: \( n = 2^{\log n} + 1 \)

To establish the new property, we are supposed to generate the index I of DNP. Here, we provide formulae for each case including the case 3.

For Case 1: \( i = 2^{\log n} - 1 + 1, \text{where} \ i > 1 \)

For Case 2: \( i = 2^{\log n} + 1 \text{ where} \ i > 1 \)

For Case S:
\[
i = 2^{\log n} - 1 + 1 \text{ and } i = 2^{\log n} + 1, \text{where} \ i > 1
\]

5. Significance of the Work:

When Pascal graph merged with Hypercube to form Simple-(q,n)-graph [2]; it showed great credentials to be exploited as a typical computer network topology. It alone can contribute substantially in this field of active research. This new discovery will earn Pascal graph high esteem through its reliability under erratic and exigent erroneous circumstances and make it sure that the situation does not exacerbate any more. As per property (iii) mentioned in some property of Pascal graph, it is very easy to access or traverse any node from v₀ directly in a single hop. Every other node except v₀ had to use either two hop (the two communicating nodes have even number indices) or single hop otherwise. Everything is fine till v₀ is not perturbed. If, under any faulty circumstances, v₀ gets disconnected then the above mentioned property no longer holds for Pascal graph and the reliability is seized with an increase in the minimum number of hops required by the message to travel from one node to another. In this paper we have reestablished its reliability under this circumstances by finding DNP that is similar to v₀ in PG(n). by this work we have eradicated threat imposed upon Pascal graph by mollifying the faulty circumstances. The superiority of the graph is maintained very efficiently.

As the whole work is not based on any convoluted formulae or theory, it is easy to exploit Pascal graphs’ high reliability, effective degree and diameter from computer network point of view. Now, it is a reliable choice for any network designer to opt the Pascal graph when reliability along with efficient degree and diameter are the key factors in designing. Since the new property is based on very simple logic it will be easier to incorporate the concept into the physical and logical layout of the computer network topology.

6. Scope of Future work:

Selecting various graph models to represent topologies of computer network had always been an active research area for network Scientists [5]. The reasonable criteria behind this selection are very significant because this put long testing impact on the network. How to make a graph model an ideal choice to implement a specific topology would always impose a huge challenge to the upcoming Scientists. The chance of finding optimal, if not optimum, graph model with unparallel properties entirely depends on the institution of the Scientists, in-depth analysis of the graph and technical feasibility of implementing computer network using that graph. Finally, it can be said very intuitively that this field still holds immense scopes for Scientists for further research work.

7. Conclusion:

A source code has been written in C programming language to generate the Pascal Matrix of different order. The formulae were put into test for a meticulous checking of the existence of DNP in different Pascal Matrices generated by the program. From the case 3, the Pascal graph is at a precise level of reliability. The Pascal Matrix has been generated along with some extra details e.g. the degree of particular node and its index. Unlike other cases, it has got three nodes that are adjacent to all other nodes.

8. References:


