RECIPROCITY AND EMC MEASUREMENTS

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Abstract: Reciprocity theorems for electrical networks and electromagnetic fields allow us to better understand the mechanisms that play a role in EMC measurements or to facilitate EMC measurements. This tutorial paper presents the theorems and their accompanying mathematical relations. Quite a number of rather simple but relevant examples demonstrate their usefulness in the field of transfer function and conversion measurements, antenna factors, radiated emission and immunity measurements, and shielding effectiveness.

1. Introduction

In 1928 the radio pioneer Stuart Ballantine wrote: "Among the tools of thought and artifices by which man forces his mind to give him more service, perhaps the most intensely useful are the simple mathematical rules of inversion known as *reciprocity theorems*" [1]. His words have not lost their meaning today. This paper aims to present reciprocity theorems useful in the field of EMC measurements in a tutorial way, that is limiting the theory to a level that allows a reasonable understanding of the relations used in the applications. We also present quite a number of rather simple applications to demonstrate the service given by these theorems.

The reciprocity theorems discussed here are in no way new and in a tutorial paper it can do no harm to mention several of the original (historical) publications, particularly since these publications offer the reader quite interesting discussions about the 'ins and outs' of these theorems. There are some famous names connected with the reciprocity theorems for use in experimental physics, in this case EMC measurements. In experimental physics, perhaps the earliest theorem is that about the reversibility of light rays, published in 1866 by Hermann von Helmholtz in his famous *Handbuch der Physiologischen Optik* [2]. This theorem springs to mind when considering the reciprocity of shielding effectiveness.

In 1877 Lord Rayleigh published his theorem dealing with electrical networks in his famous book *The Theory of Sound* [3]. This theorem is of importance when transfer functions (transfer impedance, filter attenuation, site attenuation, etc.) of linear passive networks have to be measured (Sections 2 and 3).

The reciprocity theorem for electromagnetic fields (Section 4) was formulated in 1895 by Hendrik Antoon Lorentz (Nobel Prize winner in 1905) [4, 5] and then seemingly almost forgotten for quite some time. Following Guglielmo Marconi's success in 1895 in demonstrating the possibility of sending and receiving signals using electromagnetic waves, radio communication research boomed. A reciprocity theorem for electromagnetic fields was very much needed, particularly to understand the behaviour of transmitting and receiving antennas. As 'Necessity is the Mother of Invention', John R. Carson of Bell Labs 're-invented' the theorem in 1924 [6] as did H. Pfrang in his Ph.D. thesis in Germany in 1925. Pfrang's results were used by his professor Arnold Sommerfeld in a publication in 1925 [7], where Sommerfeld also writes: "My friend M. von Laue (Nobel Prize winner in 1918, author's note) raised the surmise that this theorem could be found in the early work of H.A. Lorentz about electromagnetic waves. It indeed turned out that exactly 30 years ago Lorentz has published a beautiful and general theorem from which Pfrang's results can easily be derived." The 'connection' between the field reciprocity theorems formulated by Lorentz, Carson and Sommerfeld-Pfrang was made by Ballantine in 1928 [1].

Today, Lorentz's original publication is rather difficult to read since vector analysis had yet to be 'invented' in 1895, making his notation rather difficult to understand. In 1921, M. Abraham published his book *Theorie der Elektrizität* [8] the first chapter of which, written by A. Föppl, is devoted to vector analysis. It is most interesting to read Abraham's arguments in the introduction to his book of why vector analysis should be used in electromagnetic theory. Although the book was written in German, it is very clear from the many references made to this book that there was no language problem in the old days and, furthermore, that vector analysis was (and still is) a very useful tool, also used by Lorentz in his later publications [5]. Still, in the 1920's vector analysis was quite new and in [6] Carson writes in a foot note: "In the following proof it is necessary to assume a knowledge on the part of the reader of the elements of vector analysis; the notation is that employed by Abraham". Today the publications by Carson, Ballantine and Sommerfeld are quite readable if you bear in mind that in their days the system of units was not the same as our current system.

Ballantine, whose name lives on in the Stuart Ballantine Medal [9], published a most important application of the field reciprocity theorem [10], referring to important work carried out by Raymond M. Wilmotte [11] of the National Physics Laboratory in the U.K. (Section 5). As we will show below, this application leads to a hybrid reciprocity theorem that is of importance when considering antenna factors, radiated emission and immunity measurements, shielding effectiveness and uncertainties in EMC measurements (Section 6). Here the meaning of 'hybrid' is that, mathematically, the theorem is expressed in terms of voltage and current, on the one hand, and in electric and magnetic field components, on the other hand.

More recent literature on these reciprocity theorems can be found in [12–15], for example, where due attention is given to the mathematics. A contemporary derivation of the hybrid reciprocity theorem used in this paper can be found in [16], for example. The material presented here was published earlier in Dutch, in a series of short articles in the journal of the Dutch EMC/ESD Society [17].

2. Kirchhoff networks

This section considers quasi-stationary linear passive electrical networks that do not contain devices that make use of the properties of magnetized ferrites, such as circulators. If electric and magnetic fields play a role, their action is contained in lumped elements and/or network parameters that describe field coupling such as the mutual inductance. In other words, we consider networks that obey the two Kirchhoff laws, so that in the remainder of this paper we will refer to these networks as Kirchhoff networks. The reciprocity theorem discussed here interrelates two states of one and the same Kirchhoff network, where the states are determined by the terminations of that network.

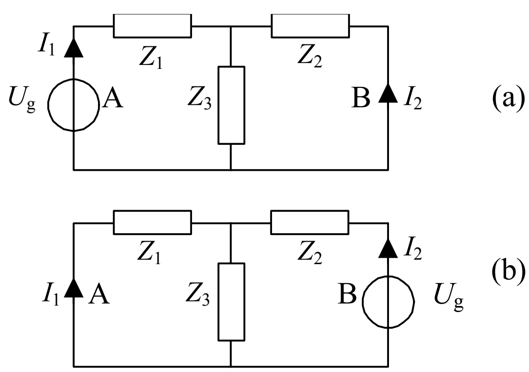

Fig.1 Network to illustrate the reciprocity theorem. (a) e.m.f. U_g at A and current measurement at B, (b) e.m.f. U_g at B and current measurement at A.

Lord Rayleigh, who formulated his reciprocity theorem rather generally in terms of forces and motions, presents various applications [3] and writes:

> "A further example may be taken from electricity. Let there be two circuits of insulated wire A and B and in their neighborhood any combination of wire circuits or solid conductors in communication with condensers. A periodic electromotive force in the circuit A will give rise to the same current in B as would be excited in A if the electromotive force operated in B".

This formulation hardly differs from that used today when introducing this reciprocity theorem, normally referring to the two circuits shown in Fig.1:

> 'If an e.m.f. U_g at the location A in a Kirchhoff network causes a current I_2^a to flow at point B in that network then a current $I_1^b = I_2^a$ will flow in point A after placing the e.m.f. U_g at the location B in that network.'

The superscripts a and b refer to the two states depicted in Figs. 1a and 1b. In more extended networks more than one e.m.f. may be present that also contribute to the current in the considered point. The theorem, however, only applies to that part of the current that is caused by the considered e.m.f.. It is very easy to verify that $I_1^b = I_2^a$ by calculating both currents. It then follows that

$$I_2^a = \frac{Z_3 U_g}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} = I_1^b \qquad (1)$$

The given formulation of the reciprocity theorem is true enough but it is not very suited for use in further considerations. The step is therefore made to the general expression that describes the reciprocity of an N-port Kirchhoff network. As proven in [6, 18]

$$\sum_{k=1}^{N} U_k^a I_k^b = \sum_{k=1}^{N} U_k^b I_k^a \qquad (2)$$

where, again, the superscripts a and b correspond to two states that are determined by the terminations of the network and can be chosen arbitrarily within the conditions that apply. Equation (2) contains combinations of the voltages in one state (a or b) and the currents in the other state (b or a). As such, Eq.(2) has little to say. It only comes alive when N and the states a and b have been chosen. Before doing so, Eq.(2) is checked (not proven), in particular because the 'recipe' used shows great similarities with the one used in the derivation of the expression describing the reciprocity theorem for electromagnetic fields (see also Eq.(24)).

The most simple network is a one-port (N= 1) formed by an impedance Z. In such a case Eq.(2) reduces to $U_1^a I_1^b = U_1^b I_1^a$ and the correctness of this equation can be verified in a rather trivial way. Assume in the a-state the voltage across Z is given by $U_1^a = Z I_1^a$, and in the b-state by $U_1^b = Z I_1^b$. The left and right hand member of the first equation are now multiplied by I_1^b so that $U_1^a I_1^b = Z I_1^a I_1^b$ and, equally, the second one by I_1^a so that $U_1^b I_1^a = Z I_1^b I_1^a$. Subtracting the second equation from the first one yields the relation being demonstrated: $U_1^a I_1^b = U_1^b I_1^a$.

To check the general expression, Eq.(2) is first written in vector/matrix form

$$[I^b]^T [U^a] = [I^a]^T [U^b] \qquad (3)$$

where the subscript T denotes the transposed matrix. In this notation, Ohm's law leads to $[U^a] = [\boldsymbol{Z}][I^a]$ in the a-state and to $[U^b] = [\boldsymbol{Z}][I^b]$ in the b-state. In a similar way as with the one-port, the first equation is multiplied by $[I^b]^T$ and the second one by $[I^a]^T$. The result of these multiplications is (note that the order of the terms is now always of importance)

$$[I^{b}]^{T}[U^{a}] = [I^{b}]^{T}[\boldsymbol{Z}][I^{a}] \qquad (4a)$$

and

$$[I^{a}]^{T}[U^{b}] = [I^{a}]^{T}[\boldsymbol{Z}][I^{b}] = \{[I^{b}]^{T}[\boldsymbol{Z}]^{T}[I^{a}]\}^{T} \qquad (4b)$$

where in Eq.(4b) we have made use of the matrix property $[\boldsymbol{X}]^T[\boldsymbol{Y}]^T[\boldsymbol{Z}]^T = \{[\boldsymbol{Z}][\boldsymbol{Y}][\boldsymbol{X}]\}^T$. If $[\boldsymbol{Z}] = [\boldsymbol{Z}]^T$, as is the case in an N-port Kirchhoff network, Eq.(3) and hence Eq.(2) directly follow after subtracting Eq.(4b) from Eq.(4a).

3. Applications (1)

This section presents four examples that connect the reciprocity theorem for Kirchhoff networks to EMC-measurements. The examples pay particular attention to the transfer impedance, filter attenuation, the conversion of a differential-mode (DM) voltage into a common-mode (CM) current and to site attenuation. In all examples Fig.2 applies and N= 2, but the applications are not limited to N= 2. For example, if cross-talk between parallel lines is considered, N= 3 or even higher may give useful information.

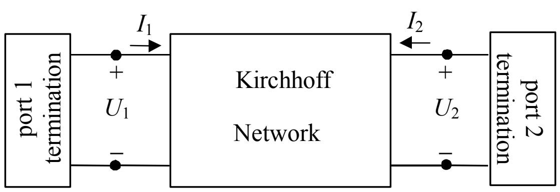

Fig.2 A two-port Kirchhoff network with a termination at each port that depends on the chosen application.

If N= 2, Eq.(2) reduces to

$$U_1^a I_1^b + U_2^a I_2^b = U_1^b I_1^a + U_2^b I_2^a \qquad (5)$$

In fact in Fig.1 N= 2 also applies. There $U_1^b = U_2^a = 0$ and $U_1^a = U_2^b = U_g$ and substitution into Eq.(5) shows again that $I_1^b = I_2^a$.

3.1 Transfer impedance

The transfer impedance is the ratio of the voltage (e.m.f.) induced in a current loop by the current in another current loop. A typical example is the cable transfer impedance that characterizes the EMC behaviour of a cable (the cable 'leakage').

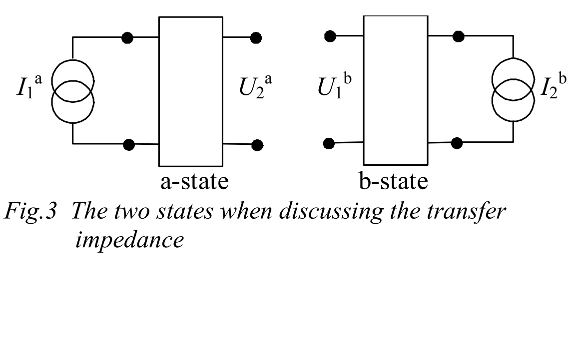

Fig.3 The two states when discussing the transfer impedance

When applying Eq.(5), the termination at port 1 (see Fig.3) in the a-state is a current source of strength I_1^a and port 2 is open circuited, i.e. $I_2^a = 0$. In the b-state, a current source I_2^b terminates port 2 while port 1 is open circuited, i.e. $I_1^b = 0$. Hence, Eq.(5) reduces to $U_2^a I_2^b = U_1^b I_1^a$ or, expressed in the transfer impedances Z_{12} and Z_{21}

$$Z_{21} = \left.\frac{U_2^a}{I_1^a}\right|_{I_2^a=0} = \left.\frac{U_1^b}{I_2^b}\right|_{I_1^b=0} = Z_{12} \qquad (6)$$

So the cable transfer impedance is reciprocal if the cable behaves as a Kirchhoff network. The cable is always passive and is always linear at most practical signal levels, as long there is no magnetic material in the cable construction. If magnetic material is used, the current (e.g. the CM current on the cable) must be verified to make sure that it is so low that no saturation of the magnetic material results. When measuring the transfer impedance it is often very difficult, if not impossible, to sufficiently satisfy the condition $I_1^b = 0$ or $I_2^a = 0$ at high frequencies, so that the measurement result has to be corrected for this non-zero current effect. Section 6.7 on interference prediction demonstrates another application of the transfer impedance concept.

3.2. Filter attenuation

In the case of filter attenuation measurements, a source (e.m.f. U_g, internal impedance Z_g) is connected via a filter to the load impedance Z_L. A well known question related to filter attenuation is: 'Does it matter which of the filter ports is connected to the source and which port is connected to the load of that source?' If the filter itself is not purely symmetrical, the EMC engineer will answer that question with 'Yes', although he or she will not be able to demonstrate this using a 50Ω measuring system (generator and voltmeter having equal impedances, e.g. $Z_g = Z_L = 50\Omega$). The latter can be verified as detailed below.

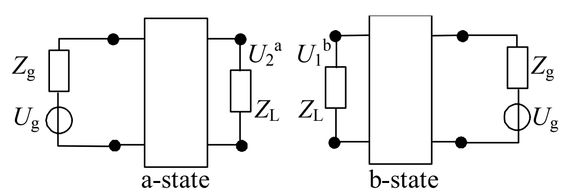

Fig.4 The two states when discussing the filter attenuation

Assume that in the a-state port 1 is terminated by the source and port 2 by the load, and the reverse termination holds in state b (see Fig.4). If U_0 is the voltage across Z_L in the absence of the filter, the filter attenuation $A^a = U_2^a/U_0$ in the a-state and $A^b = U_1^b/U_0$ in the b-state. So here it is relevant to consider the ratio $A^a/A^b = U_2^a/U_1^b$. At the terminations the following relations are valid

$$\begin{aligned} U_1^a &= U_g - I_1^a Z_g \\ U_2^b &= U_g - I_2^b Z_g \\ U_1^b &= -I_1^b Z_L \\ U_2^a &= -I_2^a Z_L \end{aligned} \qquad (7a–7d)$$

From Eqs.(7c) and (7d) it follows that $U_2^a/U_1^b = I_2^a/I_1^b$ and Eqs.(5) and (7) yield

$$\frac{A^a}{A^b} = \frac{I_2^a}{I_1^b} = \frac{I_1^a(Z_g - Z_L) - U_g}{I_2^b(Z_g - Z_L) - U_g} \qquad (8)$$

The attenuation will be independent of the choice of source port and load port if $A^a/A^b = 1$. In all other cases $A^a/A^b \neq 1$ and relative impedance values will determine the choice of the source and load port of the filter [19].

The condition $A^a/A^b = U_2^a/U_1^b = I_2^a/I_1^b = 1$ is met if $I_2^b = I_1^a$ and/or $Z_g = Z_L$. It is well known that the condition $I_2^b = I_1^a$, that simultaneously makes $I_2^a = I_1^b$, is met in the case of a purely symmetrical filter; the reciprocity theorem was not needed to demonstrate this. However, the condition $Z_g = Z_L$ resulting in $A^a/A^b = 1$ is sometimes overlooked as noted in Section 6.3. Moreover, it is this condition that explains why the engineer will measure no difference in the attenuation if source and load port are interchanged when using a 50Ω measuring system.

Of course, another path could also have been followed to arrive at the two conditions representing $A^a = A^b$. The Kirchhoff network can be characterized by a T-network and U_2^a/U_1^b can be expressed in the network impedances Z_1, Z_2 and Z_3 (see Fig.1). After straight forward calculations it then follows that

$$\frac{U_2^a}{U_1^b} = \frac{Z_2 Z_L + Z_1 Z_g + Z^2}{Z_1 Z_L + Z_2 Z_g + Z^2} \qquad (9)$$

where $Z^2 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1 + Z_3Z_g + Z_gZ_L + Z_3Z_L$. The condition $Z_1 = Z_2$ then represents the purely symmetrical filter and, again, the condition $Z_g = Z_L$ represents the 50Ω measuring system.

3.3. DM/CM conversion

The application of a reciprocity theorem may change an unsuccessful measurement into a successful one. An example is the measurement of the conversion of a differential-mode (DM) voltage into a common-mode (CM) current to characterize the emission properties of a telephone subscriber line or a power line to which a digital-signal is applied. The DM voltage U_{DM} is supplied by the digital signal, and the resulting CM current I_{CM} is a direct measure of the radiated emission capability of such a line. This measure can be expressed in a conversion admittance $Y_{CM} = I_{CM}/U_{DM}$.

To measure this conversion over a certain frequency range, e.g. 0.1 MHz – 30 MHz, it seems rather obvious to apply a DM voltage to the line and to measure the resulting CM current. However, the line is also a relatively efficient receiving antenna and ambient fields will induce CM currents that may be of the same order of magnitude as the CM currents being measured, or possibly even larger. Another problem might be the correct measurement of a DM voltage at the higher end of the frequency range. Both problems can be circumvented by applying a CM voltage U_{CM} to the line and measuring the resulting DM current I_{DM}, i.e. by determining the conversion admittance $Y_{DM} = I_{DM}/U_{CM}$. The reciprocity theorem can now be used to find the condition leading to $Y_{CM} = Y_{DM}$.

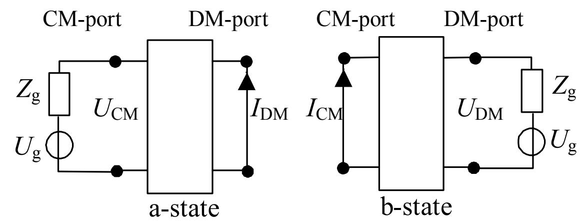

Fig.5 The two states when discussing the DM/CM conversion

The line being characterized can be considered as a two-port network where port 1 is the CM port and port 2 the DM port, see Fig.5. The following choice of terminations in the a- and b-state is now possible. In state a, representing the determination of Y_{DM}, port 1 is terminated by the voltage generator and a high impedance FET probe measures $U_1^a = U_{CM}$. Port 2 is short-circuited ($U_2^a = U_{DM} = 0$) and a current probe around the short circuit measures $I_2^a = I_{DM}$. In state b, representing the determination of Y_{CM}, port 2 is terminated by the generator and port 1 by the short circuit. In this case Eq.(5) reduces to $U_1^a I_1^b = U_2^b I_2^a$, so that

$$Y_{CM} = \left.\frac{I_{CM}}{U_{DM}}\right|_{U_{CM}=0} = \left.\frac{I_{DM}}{U_{CM}}\right|_{U_{DM}=0} = Y_{DM} \qquad (10)$$

A practical way of measuring Y_{DM} is to use Macfarlane's probe [20]. As described in [21], the CM voltage is applied to the CM input of the probe. The voltage is measured via a high impedance FET probe at this input and the current is measured in the short circuit between the probe's two DM terminals. Reference [21] also gives measured Y_{DM} data.

3.4. Site attenuation

Although only voltages and currents have been considered above, it does not mean that the reciprocity theorem does not apply if electric and magnetic fields play a role in the signal transfer. This should already be clear from the given examples, as field couplings play an important role in each of these examples.

Another example is the determination of the site attenuation in which the signal transfer between two antennas above a reflecting plane is considered. Fortunately, the signal transfer can be modelled into a passive two-port network [22] containing linear impedances as already described by Brown and King in 1934 [23]. Using a 50Ω measuring system, we can conclude from the discussion in Section 3.2 that it is not important which port is connected to the generator and which one to the receiver as long as the antennas have fixed positions. However, as noted in Section 6.2, this conclusion might not always be correct if the so-called normalized site attenuation is considered

4. Electromagnetic fields

This section considers the Lorentz reciprocity theorem interrelating the electromagnetic fields in two states that can occur in one and the same domain in space. Using Carson's formulation [24] and today's notation the reciprocity theorem for fields reads:

"If $\boldsymbol{E}^{a}$, $\boldsymbol{H}^{a}$ are the field vectors due to a periodic disturbance from a source A_1 located at O_1 and $\boldsymbol{E}^{b}$, $\boldsymbol{H}^{b}$ are the corresponding field vectors due to a disturbance originating in A_2 from a source located at O_2, then

$$\int_{1+2}(\boldsymbol{E}^{a}\times\boldsymbol{H}^{b})\cdot \mathrm{d}\boldsymbol{s}=\int_{1+2}(\boldsymbol{E}^{b}\times\boldsymbol{H}^{a})\cdot \mathrm{d}\boldsymbol{s} \quad (11)$$

the surface integrals as indicated by the subscripts 1+2 being taken over closed surfaces 1 and 2 surrounding the sources A_1 and A_2 respectively."

Not directly given in [24] is the additional equality also following from the reciprocity considerations

$$\int_{D}(\boldsymbol{E}^{a}\cdot\boldsymbol{J}^{b})\,\mathrm{d}v=\int_{D}(\boldsymbol{E}^{b}\cdot\boldsymbol{J}^{a})\,\mathrm{d}v \quad (12)$$

where D is the volume containing the sources A_1 and A_2 represented by the source vectors $\boldsymbol{J}^{a}$ and $\boldsymbol{J}^{b}$, respectively. In Eq.(11) the surface of D is indicated by the subscripts 1+2. Equation (12) will be used in the derivation of the hybrid reciprocity theorem in Section 5. As will be noted below, Eqs.(11) and (12) are connected to a special case of the general reciprocity relation Eq.(19).

So the theorem combines two field states (a and b), comparable to the discussed reciprocity theorem for an N-port Kirchhoff network. The derivation of Eqs. (11) and (12) starts from sets of two Maxwell's equations expressed in the frequency domain for the states a and b:

$$\begin{aligned}\mathrm{curl}\boldsymbol{E}^{a}&=-j\omega\mu\boldsymbol{H}^{a}\\ \mathrm{curl}\boldsymbol{H}^{a}&=+j\omega\varepsilon\boldsymbol{E}^{a}+\boldsymbol{J}^{a}\end{aligned} \quad (13a,b)$$

and

$$\begin{aligned}\mathrm{curl}\boldsymbol{E}^{b}&=-j\omega\mu\boldsymbol{H}^{b}\\ \mathrm{curl}\boldsymbol{H}^{b}&=+j\omega\varepsilon\boldsymbol{E}^{b}+\boldsymbol{J}^{b}\end{aligned} \quad (14a,b)$$

Now the trick is to remember the existence of the vector relation $\mathrm{div}(\boldsymbol{X}\times\boldsymbol{Y})=\boldsymbol{Y}\cdot\mathrm{curl}\boldsymbol{X}-\boldsymbol{X}\cdot\mathrm{curl}\boldsymbol{Y}$ and to write

$$\mathrm{div}(\boldsymbol{E}^{a}\times\boldsymbol{H}^{b})=\boldsymbol{H}^{b}\cdot\mathrm{curl}\boldsymbol{E}^{a}-\boldsymbol{E}^{a}\cdot\mathrm{curl}\boldsymbol{H}^{b} \quad (15)$$

$$\mathrm{div}(\boldsymbol{E}^{b}\times\boldsymbol{H}^{a})=\boldsymbol{H}^{a}\cdot\mathrm{curl}\boldsymbol{E}^{b}-\boldsymbol{E}^{b}\cdot\mathrm{curl}\boldsymbol{H}^{a} \quad (16)$$

An expression for $\boldsymbol{H}^{b}\cdot\mathrm{curl}\boldsymbol{E}^{a}$ in Eq.(15) can be found by multiplying the left and right hand members of Eq.(13a) by $\boldsymbol{H}^{b}$ (be careful with the order of the variables)

$$\boldsymbol{H}^{b}\cdot\mathrm{curl}\boldsymbol{E}^{a}=-j\omega\mu\boldsymbol{H}^{b}\cdot\boldsymbol{H}^{a} \quad (17)$$

In a similar way, expressions can be found for the other terms on the right hand side of Eqs.(15) and (16) by multiplying Eqs.(13b), (14a) and (14b) by the proper vectors. After substitution of all resulting expressions in Eqs.(15) and (16) and subtracting the two final equations, we find that

$$\mathrm{div}\left(\boldsymbol{E}^{a}\times\boldsymbol{H}^{b}-\boldsymbol{E}^{b}\times\boldsymbol{H}^{a}\right)=\boldsymbol{E}^{b}\cdot\boldsymbol{J}^{a}-\boldsymbol{E}^{a}\cdot\boldsymbol{J}^{b} \quad (18)$$

This equation, in which the ω-terms no longer appear, is sometimes called the local form of the reciprocity theorem. To find the general form, Eq.(18) has to be integrated over the volume D containing all sources represented by $\boldsymbol{J}^{a}$ and $\boldsymbol{J}^{b}$, yielding

$$\begin{aligned}&\int_{D}\mathrm{div}\left(\boldsymbol{E}^{a}\times\boldsymbol{H}^{b}-\boldsymbol{E}^{b}\times\boldsymbol{H}^{a}\right)\cdot\mathrm{d}\boldsymbol{v}=\\ &\int_{S}\left(\boldsymbol{E}^{a}\times\boldsymbol{H}^{b}-\boldsymbol{E}^{b}\times\boldsymbol{H}^{a}\right)\cdot\mathrm{d}\boldsymbol{s}=\\ &\int_{D}\left(\boldsymbol{E}^{b}\cdot\boldsymbol{J}^{a}-\boldsymbol{E}^{a}\cdot\boldsymbol{J}^{b}\right)\cdot\mathrm{d}\boldsymbol{v}\end{aligned} \quad (19)$$

We had to expect an integration as the Maxwell equations consider field derivatives whereas an expression for the fields is needed. The most left hand member of Eq.(19) has been converted from an integral over a volume D into an integral over the surface S of D by applying Gauss's theorem. This action also 'removes' the div operation. Equation (19) is the general form of the Lorentz reciprocity theorem, a form that can even be extended [14], although this extension is not needed in the context of this paper.

A special case is the situation in which the relation between the E and the H vector of the field is fixed, such as in the far-field of an antenna. Then the outcome of the integrals in Eq.(19) is equal to zero, a value given by Lorentz as he considered the propagation properties of light. In the far-field, the field propagates as a plane or quasi-plane wave so that the E and the H vector of the field are perpendicular to each other and have a constant ratio, $\eta=\sqrt{(\mu/\varepsilon)}$, the wave impedance (377 Ω in air). In vector notation the latter means that $\boldsymbol{\nu}\times\boldsymbol{E}=\eta\boldsymbol{H}$, where $\boldsymbol{\nu}$ is the unit vector perpendicular to the plane formed by $\boldsymbol{E}$ and $\boldsymbol{H}$. After application of this relation to $\boldsymbol{E}^{a}\times\boldsymbol{H}^{b}$ and to $\boldsymbol{E}^{b}\times\boldsymbol{H}^{a}$ and after application of the vector relation $\boldsymbol{X}\times(\boldsymbol{Y}\times\boldsymbol{Z})=\boldsymbol{Y}\cdot(\boldsymbol{X}\cdot\boldsymbol{Z})-\boldsymbol{Z}(\boldsymbol{X}\cdot\boldsymbol{Y})$ we find that

$$\boldsymbol{E}^{a}\times\boldsymbol{H}^{b}=\boldsymbol{E}^{b}\times\boldsymbol{H}^{a}=\eta(\boldsymbol{E}^{b}\cdot\boldsymbol{E}^{a}) \quad (20)$$

so that $\boldsymbol{E}^{a}\times\boldsymbol{H}^{b}-\boldsymbol{E}^{b}\times\boldsymbol{H}^{a}=0$ and, consequently, the integrals in Eq.(19) are equal to zero and Eqs.(11) and (12) given at the start of this section automatically follow.

5. The hybrid reciprocity theorem

The experimentalist can only humbly lift his hat at the mathematical fireworks presented in Section 4 and then pass to the order of the day. An application is what is needed to catch his or her interest. As mentioned in the introduction, a very useful application was already given in 1929 by Ballantine [10], referring to the work of Wilmotte [11].This application, resulting in the hybrid reciprocity theorem, can also be found in Section 4.5 of a more recent textbook [16]. The theorem gives an expression for the voltage U_i induced by an incident field E^{i} in an antenna or structure acting as an antenna, e.g. an EUT (Equipment Under Test) and its connected cables. Since the transmission and reception of electromagnetic waves is of interest in EMC field measurements, the two states a and b will now be denoted by t (of transmission) and r (of reception).

In the following, two wire antennas A_1 and A_2 are considered (see Fig.6). Each wire antenna consists of

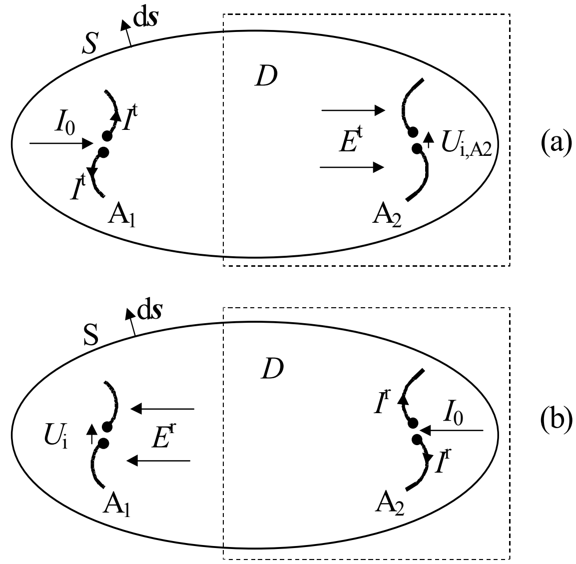

Fig.6 The states (a) transmission and (b) reception. The dashed lines indicate parts that need not necessarily be accessible.

two wire elements, separated by a small gap of width w, that determines the distance between the two terminals (each connected to a wire element) of the antenna. The wire antennas of total length L_1 and L_2 are assumed to be electrically thin and perfectly conducting. The width $w<<\{L_1, L_2\}$ is so small that a quasi-stationary approach is allowed at the gap, i.e. it is assumed that the time derivative in Faraday's law can be taken equal to zero. In other words: the wire elements are connected to a Kirchhoff network.

Figure 6a depicts the t-state: A_1 is the transmitting antenna to which a current source of strength I_0 is connected. This results in a current distribution I^t over A_1, and an incident field E^t at the location of A_2. The open circuit voltage induced in the latter antenna is $U_{i,A2}$. The r-state is considered in Fig.6b: the current source I_0 is connected to the terminals of A_2, creating a E^r at the location of A_1, in which antenna an open circuit voltage U_i is induced. The task now is to derive an expression for U_i.

If we realize that within the volume D in Eq.(12) the current can only flow on the wire elements, $\boldsymbol{J}\mathrm{d}v$ in that equation can be replaced by $I(l)\mathrm{d}\boldsymbol{l}$. The current $I(l)$ is the current (perfect conductor) uniformly distributed over the infinitesimal wire segment $\mathrm{d}\boldsymbol{l}$ at the position l along the wire element. The segment has a certain orientation with respect to the incident field, so $\mathrm{d}\boldsymbol{l}$ is a vector. Moreover, the volume integral in Eq.(12) now changes into a line integral, so that

$$\int_{L_1+w} I^{\mathrm{t}}(l)\boldsymbol{E}^{\mathrm{r}}\cdot \mathrm{d}\boldsymbol{l} = \int_{L_2+w} I^{\mathrm{r}}(l)\boldsymbol{E}^{\mathrm{t}}\cdot \mathrm{d}\boldsymbol{l} = 0 \qquad (21)$$

The outcome of the integrals equals zero because either the current is zero (in the gap) or the dot product $\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l}$ is zero because the tangential field on a perfect conductor is always zero. In such a conductor the internal E-field is always zero and the boundary condition states that also the E-field just outside the conductor is zero. In other words, at the surface of the wire element, the component of the incident field parallel to that element contributing to U_i, i.e. $\boldsymbol{E}^{\mathrm{i}}\mathrm{d}\boldsymbol{l}$, is always cancelled by the reflected or scattered field $\boldsymbol{E}^{\mathrm{s}}$ where $\boldsymbol{E}^{\mathrm{s}}\mathrm{d}\boldsymbol{l} = -\boldsymbol{E}^{\mathrm{i}}\mathrm{d}\boldsymbol{l}$. So, if all elementary wire segments were replaced by elementary sources $\boldsymbol{E}^{\mathrm{s}}\cdot\mathrm{d}\boldsymbol{l}$, the total field distribution outside the wire elements would not change [16]. In the assumed quasi-stationary situation, the voltage U^{i} is equal to the line integral of the field over the gap w, while in the t-state the gap current equals I_0. Using these results, the left hand member of Eq.(21) can be written as

$$\begin{aligned} &I_0 U_{\mathrm{i}} + \int_{L_1} I^{\mathrm{t}}(l)\boldsymbol{E}^{\mathrm{s}}\cdot \mathrm{d}\boldsymbol{l} = \\ &= I_0 U_{\mathrm{i}} - \int_{L_1} I^{\mathrm{t}}(l)\boldsymbol{E}^{\mathrm{i}}\cdot \mathrm{d}\boldsymbol{l} = 0 \end{aligned} \qquad (22)$$

so that

$$I_0 U_{\mathrm{i}} = \int_{L_1} I^{\mathrm{t}}(l)\boldsymbol{E}^{\mathrm{i}}\cdot \mathrm{d}\boldsymbol{l} \qquad (23)$$

and this relation derived from the Lorentz reciprocity theorem clearly demonstrates its hybrid character. In a general approach, it can be shown that under certain conditions the reciprocity theorem for the Kirchhoff networks, Eq.(2) can be derived from the Lorentz reciprocity theorem and the following relation holds [14, 15]

$$\begin{aligned} &\int_S \left(\boldsymbol{E}^{\mathrm{a}}\times\boldsymbol{H}^{\mathrm{b}} - \boldsymbol{E}^{\mathrm{b}}\times\boldsymbol{H}^{\mathrm{a}}\right)\cdot \mathrm{d}\boldsymbol{s} = \\ &\sum_{\mathrm{k}=1}^{\mathrm{N}} (U_{\mathrm{k}}^{\mathrm{a}} I_{\mathrm{k}}^{\mathrm{b}} - U_{\mathrm{k}}^{\mathrm{b}} I_{\mathrm{k}}^{\mathrm{a}}) = 0 \end{aligned} \qquad (24)$$

This means that there is a direct link between the Maxwell equations and the reciprocity theorem for Kirchhoff networks and that we no longer have to think in terms of electrical forces and motions, as was common in Rayleigh's days.

Some general observations can be made from Eq.(23):

a) Of the fields, only the *incident* field counts!
b) The equation only contains parameters that apply to antenna A_1 or to the location of A_1. No information is needed about 'where is antenna A_2 causing the incident field E^{i}' or about 'what happens in antenna A_2'. This is indicated schematically in Fig.6 by the dashed lines.
c) Up to this point we have tacitly assumed that all ports of a network are accessible for (simultaneous) connection of terminations (of measuring equipment). Equation (23) can look at a situation in which only one port is accessible.
d) The equation meets the demands of an experiment-alist who is never able to directly measure the field strength. He or she always needs a conversion of a field strength quantity into a quantity that can be measured via conduction.

e) Equation (23) can also be written as

$$U_{\mathrm{i}} = \int_{L_1} \frac{I^{\mathrm{t}}(l)}{I_0} \boldsymbol{E}^{\mathrm{i}} \cdot \mathrm{d}\,\boldsymbol{l} \qquad (25)$$

so that $I_t(l)/I_0$ can be considered to be the normalized current distribution in the transmitting state that weighs the contributions of incident field in the receiving state.

6. Applications (2)

This section presents a number of simple applications of Eq.(23) or Eq.(25), dealing with the antenna factor, radiated immunity measurements and shielding. Interesting and rigorously treated applications of Eq.(24) can be found in [15, 25]. Only in simple cases can the integral in Eq.(23) or Eq.(25) be solved analytically; an example is given in Section 6.1. However, the other examples will show that quite useful information is made available without solving the integral.

6.1. The λ/2-dipole

A very simple application of Eq.(25) follows if we calculate the voltage induced in a λ/2-dipole in free space. As can be found in all current text books on antennas and was verified experimentally by Wilmotte in 1927 [26], the current distribution in the transmitting state of this antenna is half a sine wave. If the incident field is a plane wave of strength E^{i} with its polarization parallel to the wire elements, the well known expression for the induced voltage follows from Eq.(25):

$$U_{\mathrm{i}} = \frac{E^{\mathrm{i}}}{I_0} \int_{-\lambda/4}^{\lambda/4} I_0 \sin\!\left(\frac{2\pi}{\lambda}\left(\frac{\lambda}{4} - l\right)\right) \mathrm{d}\,l = \frac{\lambda}{\pi} E^{\mathrm{i}} \qquad (26)$$

If Z_{m} is the effective load impedance and Z_{a} the internal impedance of the λ/2-dipole antenna, the voltage U_{m} measured by the receiver is given by

$$U_{\mathrm{m}} = \frac{Z_{\mathrm{m}}}{Z_{\mathrm{m}} + Z_{\mathrm{a}}} \frac{\lambda}{\pi} E^{\mathrm{i}} \qquad (27)$$

and if $E^{\mathrm{i}} = E_{\mathrm{c}}^{\mathrm{i}}$ is the field strength during calibration of this antenna, its antenna factor F_{A} is given by

$$F_{\mathrm{A}} = \frac{\pi (Z_{\mathrm{m}} + Z_{\mathrm{a}})}{\lambda Z_{\mathrm{m}}} \qquad (28)$$

By stating 'Z_{m} is the effective load impedance of the antenna' and not 'Z_{m} is the input impedance of the receiver', the properties of the antenna balun are assumed to be taken into account.

Seibersdorf, for example, is a supplier of a set of 27 λ/2-dipole antennas suitable for performing the validation of an antenna calibration test site as described by CISPR/A [27]. For these antennas $Z_{\mathrm{m}} = 100\ \Omega$, and the free space value of $Z_{\mathrm{a}} = 73\ \Omega$. Inserting these values in Eq.(28) results in F_{A} values that differ less than 0.1 dB from the theoretical values supplied by Seibersdorf (maybe, Seibersdorf also used Eq.(28)).

6.2. The antenna factor F_{A}

This section should clarify which parameters play a role in the determination of the antenna factor and what the consequences are in radiated emission measurements, measurement uncertainty and normalized site attenuation measurements.

By definition, the measured field strength $E_{\mathrm{m}} = F_{\mathrm{A}} U_{\mathrm{m}}$, so the antenna factor can be written as

$$F_{\mathrm{A}} = \frac{Z_{\mathrm{m}} + Z_{\mathrm{a,c}}}{Z_{\mathrm{m}}} \frac{1}{\int_L (I_{\mathrm{c}}^{\mathrm{t}}(l)/I_0) \boldsymbol{E}_{\mathrm{c}}^{\mathrm{i}} \cdot \mathrm{d}\boldsymbol{l}} \qquad (29)$$

The antenna factor therefore depends on 3 variables determined by the *calibration set-up*, in Eq.(29) indicated by the subscript c,

1) The antenna impedance $Z_{\mathrm{a,c}}$,
2) The normalized current distribution $I_{\mathrm{c}}^{\mathrm{t}}(l)/I_0$, and
3) The outcome of the dot-product $\boldsymbol{E}_{\mathrm{c}}^{\mathrm{i}} \cdot \mathrm{d}\boldsymbol{l}$ along the wire elements.

As a consequence, if in a radiated emission measurement one or more of these three variables differ from the values during calibration, the antenna factor is *unknown*. In discussions about the standardization of radiated emission tests, the first variable (the antenna impedance) has often been considered, but not the other two variables.

The normalized current distribution depends on the interaction of the antenna with its environment during its actual use (calibration or radiated emission test). Of course, the outcome of the integral depends on the incident-field distribution which, however, is not specified in a radiated emission compliance test. The wire elements of a receiving antenna automatically 'integrate' over the field distribution whether the measurement is carried out on an open area test site, or in a semi or a fully anechoic room. Consequently, antennas with different shapes will have different induced voltages, even if the incident field is a plane wave.

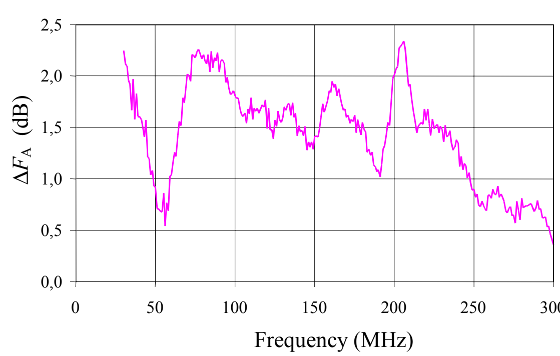

Fig 7. The difference $\Delta F_A = F_A$(ANSI/3m)$-F_A$(Free-Space) between the antenna factors of a log-biconical antenna derived from the two calibration reports

The combined effect of all three variables comes to the fore in the first example. Figure 7 shows the difference ΔF_{A} between the ANSI(3m) antenna factor and the free-space antenna factor of one and the same log-biconical antenna, as determined by a UKAS accredited company.

We can also conclude from the above that the linearized model used in documents on EMC measurement instrumentation uncertainties [28, 29] cannot be justified. Moreover, the uncertainty in the antenna factor during calibration has little in common with the uncertainty in the antenna factor during an actual radiated emission test. In the normalized site attenuation measurement method the result of the site attenuation measurement is normalized to the antenna factors, after which the result is compared to a theoretically predicted result. This method is formally only correct if it can be demonstrated that the antenna factors used are valid during the conditions of the site attenuation measurement.

As a second example, Fig.8 shows the difference dE_{max} when in the CISPR/A radiated emission round robin test (RRT) the field-strength emitted by the battery operated tightly specified EUT (a rod antenna above a small ground plane) was firstly measured using a biconical antenna and secondly by using a log-biconical antenna [30]. When replacing the receiving antenna, care was taken that the remaining part of the set-up was not changed. Only the prescribed measurement distance was adjusted by moving the receiving antenna. In this example, the effect of integrating over different parts of the incident field distribution comes to the fore very clear. This effect was also found in the statistical evaluation of the RRT field strength measurement results using the tightly specified EUT [30].

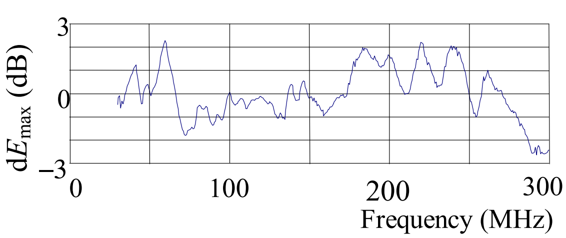

Fig.8 Measured field strength difference dE_{max} (dB) after a biconical antenna has been replaced by a log-biconical antenna

When using a log-biconical antenna, the effective measurement distance changes with frequency, and it has sometimes been suggested that is possible to correct for this effect. From the theory above, it will be clear that such a correction is only possible if the complete actual incident-field distribution is known and accounted for in the correction factor.

6.3. Probe calibration in a TEM cell

Field probes are often calibrated in a TEM-cell. However, the practical use of these probes is generally outside such a cell. So the current distribution in the t-state, and, consequently, the antenna factor, may be different in cases where the probe is not as close to the metal plates as it is inside the TEM-cell. It seems that this aspect was overlooked in [31]. It might therefore be one of the reasons for the limited agreement between measurement results obtained in the TEM cell and those obtained outside that cell. In addition, the authors in [31] claim that the reciprocity of the TEM cell was verified experimentally. In that experiment, a first transfer function was measured after connecting the signal generator to the probe acting as transmitting antenna inside the cell and the measuring receiver to one of the cell terminals. A second transfer function was measured after reversing the connections, i.e. after connecting the generator to the cell terminal and the measuring receiver to the probe. They then conclude that reciprocity has been demonstrated as the two transfer functions differed by less than 1 dB. However, for trivial reasons the TEM cell was a linear passive device and, hence, was reciprocal. The discussions in Section 3.2 then indicate that the experiment only demonstrated that the ratio of the output impedance of the generator and the input impedance of the measuring receiver was smaller than 1 dB. If a 50Ω generator and a 75Ω measuring receiver would have been used, for example, that ratio would most likely have been different and the authors would have discovered that their method needed an additional consideration.

Another application of the reciprocity theorem involving a TEM cell is given in [32].

6.4. Radiated immunity, E^i

In a radiated immunity test, it is the induced voltage that may cause malfunctioning of the EUT. Equation (25) clearly indicates that this voltage is determined by the incident field *and* by the normalized current distribution. In this section, aspects of the incident field are considered and in Section 6.5 aspects of the current distribution are considered.

The incident field is the field that would be present in absence of the EUT plus its attached cables acting as an antenna. Consequently, the specified field strength in a radiated immunity test is normally measured and adjusted before the placement of the EUT using a small probe (negligible interaction with the field source). It is not correct to measure the specified field strength using a small probe near the EUT, because that probe measures the field *incident to the probe*. The latter field may significantly differ from the incident field experienced by the EUT, as it is the combination of the wanted field and the field reflected from the EUT. The field incident to the probe might even be almost zero if the desired test field and the reflected field are in anti-phase.

After placement of the EUT we need to verify that the incident field as such has not changed as a result of a possible strong interaction between the EUT and the source of the field. Such an effect may be observed by comparing the forward power measured via a directional coupler, in the connection between the generator and antenna emitting the test field during the previously mentioned field adjustment, with the power measured after the placement of the EUT. If the forward power has changed, the desired incident field has changed. A first correction is to adjust the generator output to a level that results in the original forward power. However, from the hybrid theorem it follows that this adjustment does not need to be 100% correct, as the field distribution during the field adjustment and that after placement of the EUT need not be the same.

6.5. Radiated immunity, $I^t(l)/I_0$

Equation (25) also indicates that the normalized current distribution in the t-state is of importance. Since that distribution is the weighting function of the voltage contributions induced by the incident field, resonances in that distribution may be noticed in the disturbance signal induced in the cable attached to an EUT. An example is given in Fig.9, where the maximum (max), average (avg) and the minimum (min) value of the measured induced CM-current are plotted as a function of the frequency of the homogeneous incident field with a strength of 1 V/m. Eight EUTs taken from a class of electrically small EUTs were tested [21].

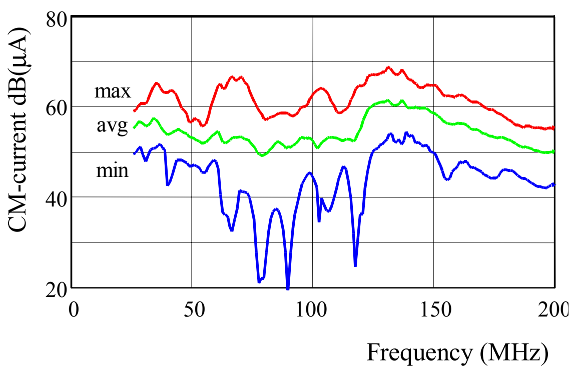

Fig.9 The induced common-mode current measured close to an electrically small EUT in the cable attached to that EUT, when illuminated by a field of 1 V/m (8 EUTs)

Although the average value is about 55 dBμA (a value close to the rule of thumb for these EUTs: 1 mA per V/m), the curve labelled 'min' indicates that resonances may cause a minimum in the induced current, so that the considered EUT is hardly tested for immunity around the resonance frequencies. In other words, a uniform and constant incident field does not guarantee that the EUT is tested with a constant actual disturbance signal (here represented by the CM current). In addition, we should expect the resonance frequencies to shift when the layout of the cables is changed. If it had been possible to let the EUT act as a transmitter, the resonances would also have been found, comparable to the resonances of a rod antenna.

In the case of an interference complaint in which a product is insufficiently immune to EM fields, it is not always possible to solve the problem at the location where the product is used. The disturbance field strength at that location is measured and the product is taken to the test lab to carry out a radiated immunity test with that field strength. However, if the CM current distribution on the cables attached to that product differs significantly in the test situation from that at the complaint location, the test might not cover the actual complaint. So it is advisable to measure not only the field strength at the complaint location, but also the CM current on the attached cables (close to the product) and to verify whether these currents are (more or less) the same in the test house.

6.6. Shielding

This section addresses the frequently asked question 'If a shield attenuates the field emanating from circuits inside that shield by an amount of X dB, are these circuits then also shielded by an amount of X dB for fields generated outside that shield?' We can illustrate the reasoning behind the answer by the following rather simple configuration, that allows the use of simple analytical relations. Rigorous approaches based on the Lorentz theorem can be found in [25, 33]. In Section 6.7 the results are also used in an example of interference prediction.

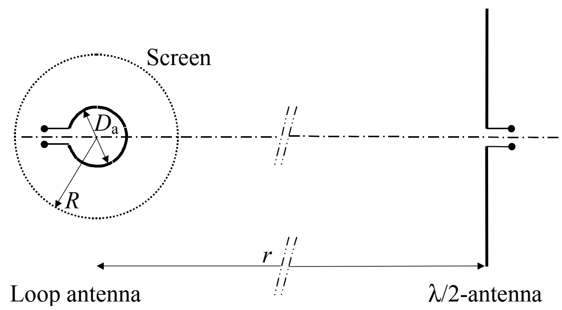

Fig.10 Schematic drawing for use in the application of the reciprocity theorems

A tuned λ/2 dipole is located at a distance r in the far-field of a small loop antenna, as shown in Fig.10. Both antennas are located in free space so that antenna coupling and reflections do not play a role. A signal source $\{U_g, R_g\}$ can be connected to the loop antenna, and a voltmeter (input impedance $R_v = R_g$) can be connected to the λ/2 dipole (internal impedance Z_a), and vice versa. The area of the loop antenna $A = \pi D_a^2/4$ and the internal impedance of that antenna is Z_l. The orientation of both antennas is such that an optimal signal transfer results.

If the source is connected to the loop antenna and the voltmeter to the λ/2 dipole, the voltmeter reading U_{v1} in the absence of the screen will be given by

$$U_{v1} = \frac{\lambda R_g}{\pi(R_g + Z_a)} \cdot \frac{Z_0 k^2 A U_g}{4\pi r (R_g + Z)} \qquad (30)$$

where $\lambda R_g/\{\pi(R_g+Z_a)\}$ is the voltmeter reading in the case of an incident field given by the second part of the right hand member of Eq.(30), (also see Eq. (27)). In that part, $AU_g/(R_g+Z_l)$ is the magnetic dipole moment of the loop antenna, Z_0 the far-field wave impedance and $k = 2\pi/\lambda$. Next, a spherical screen of radius R, $D_a/2 << R << \{r, \lambda/2\pi\}$, is put around the loop antenna such that the loop is in its center. The screen is assumed to be in the near field of the loop. Now the voltmeter reading is U_{v2} and, consequently, the shielding effectiveness $S_H = U_{v1}/U_{v2}$. Because the sphere also acts as a magnetic dipole [25], U_{v2} is also described by

Eq.(30), although the magnetic dipole moment is now a factor S_H smaller.

The next step is to connect the generator to the λ/2 dipole and the voltmeter to the loop antenna. In the absence of the screen U_{v3} is measured and in the presence of that screen U_{v4} is measured. The voltage U_{v3} is given by

$$U_{v3} = \frac{\mu_0 \omega A R_g}{(R_g + Z)} \cdot \frac{2U_g}{4\pi r(R_g + Z_a)} \qquad (31)$$

where $R_g/(R_g+Z_l)$ gives the voltage division of the voltage $\mu_0\omega AH$ induced by the incident H-field. That field is given by the second part of the right hand member of Eq.(31), and is easily understood when remembering that in the far-field the E-field of a center fed tuned λ/2 dipole is given by $E= 60I/r= 2Z_0/(4\pi r)$ so that $H= 2I/(4\pi r)$ and where I is the current entering the antenna.

Using the relations $Z_0= \sqrt{(\mu_0/\varepsilon_0)}$ and $f{\cdot}\lambda= c= 1/\sqrt{(\mu_0\varepsilon_0)}$ we can easily verify that $U_{v1}= U_{v3}$. This is not a surprising result, since the equivalent network between the ports of the two antennas is a Kirchhoff network, which means that the associated reciprocity theorem directly gives the answer $U_{v1}= U_{v3}$ (see Eq.(8)) using $Z_g= Z_L= R_g$. However, because the screen is also linear and passive, the same theorem shows that $U_{v2}= U_{v4}$, and a simple calculation like the one above to demonstrate this result is not possible. The last statement is particularly true because the dipole field arrives as a plane wave at the screen, while the screen is in the near-field region of the loop antenna. A knowledgeable in the theory may be able to show that the Helmholtz theorem about the reversibility of light rays [2] is applicable in the described situation.

In this example the actual shielding effectiveness is determined by that of the screen in the near-field of the loop antenna. This effectiveness is generally much lower, e.g. 40 dB, than that for a plane wave generated outside the screen. The example stresses the fact that the shielding effectiveness is not entirely a property of the shield. It is a property of the shield plus the antennas or antenna structures playing a role in the disturbance signal transfer.

In conclusion, the reciprocity theorems give conditions under which the shielding effectiveness is reciprocal, and the results are certainly applicable in the case of in-band interference. In the case of out-of-band disturbances acting on non-linear devices such as transistors, the theorems are not formally applicable. However, it is still possible to follow the given path to estimate the magnitude of the induced signals and to consider the consequences of those signals afterwards [33].

6.7. Interference prediction

The results obtained in Section 6.6 can be applied to interference prediction. As an example, the following application considers the unwanted signal induced by a distant broadcasting transmitter in the antenna of Magnetic Resonance Imaging (MRI) equipment used in hospitals.

In the early days of the use of MRI equipment, hospitals did not like to have large Faraday cages around the equipment. As broadcasting transmitters could emit strong fields at the in-band frequencies of the MRI equipment, the following question arose 'Is it possible to carry out field strength measurements at the location where the MRI equipment is planned to be used *before* the placement of that equipment and to predict the level of the disturbance signal induced in the MRI antenna?' The answer was 'Yes, and with a reasonable degree of confidence'. The following three steps had to be followed to find the answer. Figure 10 is again applicable: the MRI antenna is the loop antenna and the λ/2 dipole is the antenna of the broadcasting transmitter, (normally a λ/4 antenna above the earth acting as a ground plane). Simple mathematical relations illustrate the estimate of the maximum voltage $U_{i,max}$ that could be induced in the loop antenna.

Step 1: Connect a signal source $\{U_g, R_g\}$ to the MRI antenna located at its normal-use position inside the MRI equipment, so that all interactions are properly taken into account. Set the frequency of this source to that of the (strong) broadcasting field to be expected in the hospital. Measure at a distance r_1 in the far-field of the MRI equipment the field pattern $E_1(\varphi)$, $0°\leq\varphi\leq360°$, emitted by the MRI antenna and measure the current I_0^t flowing into the loop antenna. This is a measurement that can be carried out on the manufacturers premises!

Step 2: Determine the maximum E_{max} of $E_1(\varphi)$ and assume that the MRI equipment emits this field in the direction of the λ/2 dipole at a distance r from the equipment (worst case). This field is proportional to I_0^t, so $E_{max}= \alpha_{max}I_0^t$ and the incident field for the λ/2 dipole $E^t= (\alpha r_1 I_0^t)/r$. Application of the hybrid reciprocity theorem then gives the voltage induced in the λ/2 dipole: $U_{i,\lambda/2}= (\alpha_{max}\lambda r_1 I_0^t)/(\pi r)$, (see Eq.(26)). Consequently, the transfer impedance Z_{tr} between the loop antenna and the λ/2 dipole is given by

$$Z_{tr} = \frac{\alpha_{max}\lambda r_1}{\pi r} \qquad (32)$$

and in the given situation Z_{tr} is reciprocal.

Step 3: During operation of the broadcasting transmitter, the input current to its antenna is I_0^r and that current can be determined from the field strength E^r measured at the location where the MRI equipment is to be installed, by using the well known relation $E^r= 60I_0^r/r$. Using Eq.(32), the estimate of the maximum voltage $U_{i,max}$ induced by the broadcasting transmitter in the loop antenna is given by

$$U_{i,max} = Z_{tr} I_0^r = \frac{\alpha_{max}\lambda r_1}{60\pi} E^r \qquad (33)$$

By comparing this voltage with the level allowed to operate the MRI equipment sufficiently free of interference, a decision can be made about whether or not a Faraday cage is needed and, if it is, how much attenuation that cage should present.

Summary

In this paper we have discussed the reciprocity theorem interrelating two states of one and the same Kirchhoff network (linear passive network) as determined by the terminations of that network. We have given applications improving the understanding of transfer impedance, filter and site attenuation measurements. In addition, we have used the theorem to facilitate a DM voltage to CM current conversion measurement.

We discussed the reciprocity theorem interrelating the electromagnetic fields in two states that can occur in one and the same domain in space, and from this theorem we derived the hybrid reciprocity theorem. The latter theorem was applied to a tuned $\lambda/2$ dipole, to the measurement of antenna factors, to probe calibration in a TEM cell and the uncertainties associated with these measurements. We also used the hybrid reciprocity theorem to discuss aspects of radiated immunity measurements.

Finally, we used the reciprocity the reciprocity theorems in a discussion about the reciprocity of the shielding effectiveness and in a simple method to estimate the interference potential of a field (at in-band frequencies) incident on an antenna.

Acknowledgements

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